

### Homework #4 of Topology II

Due Date: Feb 28, 2018

1. Let  $X$  be the union of the unit sphere in 3-space with the straight line segment from the north pole to the south pole. Find  $\pi_1(X)$ .
2. Let  $X$  be the union of the unit sphere in 3-space with the unit disk in the  $xy$ -plane. Find  $\pi_1(X)$ .
3. Let  $X$  be the quotient space of  $D^2$  obtained by identifying points on the boundary that are  $120^\circ$  apart. Find  $\pi_1(X)$ .
4. Let  $X$  be the quotient space of an annulus obtained by identifying antipodal points on the outer circle and identifying points on the inner circle which are  $120^\circ$  apart. Find  $\pi_1(X)$ .
5. Let  $X \subset \mathbb{R}^m$  be the union of convex open subsets  $X_1, X_2, \dots, X_n$  such that  $X_i \cap X_j \cap X_k \neq \emptyset$  for all  $i, j, k$ . Show that  $X$  is simply connected.
6. Show that the complement of a finite set of points in  $\mathbb{R}^n$  is simply connected if  $n \geq 3$ .
7. Let  $X \subset \mathbb{R}^3$  be the union of  $n$  lines through the origin. Compute the group  $\pi_1(\mathbb{R}^3 - X)$ .